A New Equivalent Circuit for the Impedance of Short Radiators

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For a long time, the series combination of a capacitance C and a radiation resistance R_S serves as the equivalent circuit for the impedance of a short monopole or dipole (**Fig 1a**). According to the theory of Hertz [1], the radiation resistance of a lossless monopole over a conductive plane (**Fig 2**) is

$$R_S = 160\,\pi^2\,(h_{eff} / \lambda_0)^2 \quad [\Omega]. \tag{1}$$

The effective antenna height h_{eff} theoretically equals about half the antenna height, but for real antennas it is subject to relatively complex laws and strongly depends on the shape of the feedpoint and the conducting plane as well as on conductive or dielectric materials in the vicinity of the antenna. We believe that some of these influences are easier to discover if a better divided equivalent circuit according to **Fig 1b** is used. Here a parallel capacitance C_1 is extracted from capacitance C of **Fig 1a** which is called *dead capacitance* and which does not contribute to antenna radiation. It remains a much smaller capacitance C_2 in series with a new and larger radiation resistance R_{S0} which describes the infinite space as an absorber for the radiated wave. The antenna is coupled to that space by capacitance C_2 which we call *space capacitance*. Interesting is our finding that, with a physically sound definition of C_2, the "imaginary" real resistance R_{S0} of space is largely independent of frequency as well as of the length and diameter of the antenna rod.

In paragraph 1 the physical explanation for the new equivalent circuit will be given and in a second paragraph it's frequency dependence will be described. In paragraph 3 this equivalent circuit will also be used for a receiving antenna with a voltage source U_0 connected in series with the radiation resistance R_{S0} according to **Fig 1c**.

1. The Division of Antenna Capacitance into Dead Capacitance and Space Capacitance

Our considerations are based on the time-dependent course of wave detachment in the near field of a rod antenna which has been calculated by Landstorfer [2] and which is also available as a motion picture [3]. In **Fig 2** a generator feeds a rod through a coaxial transmission line, during each half cycle it charges and subsequently discharges the rod in order to charge it again with reversed polarity. The rod is charged during the first quarter cycle. When the generator voltage is at maximum, the rod also carries maximum charge if it is short enough, i.e. if it's inductance is negligible. For this moment **Fig 2** shows the electric field lines with the charges at their ends [2]. Now the rod is currentless and the energy supplied by the generator is almost completely stored as electric energy in the field of the rod. There is only some small residual amount of magnetic energy in space, especially close to the conductive plane, because the charges on the plane do not come to rest but still are moving away from the rod. That magnetic energy is without significance for the impedance of the short radiator.

When the generator voltage decreases during the second quarter cycle, the rod discharges. During discharge, the energy stored in the field in the state of **Fig 2** is divided into three parts, which are delimited by the broken lines. The field lines in region I migrate back to the generator together with the charges at their ends. The electric field energy assigned to these field lines also flows back into the generator, or at least towards the feedpoint of the rod, but not out into space. So the energy which fills region I of **Fig 2** at the moment of maximum charge is only temporarily stored there and returned afterwards. Thus the generator delivers reactive power and it is loaded by that region I as by the capacitance C_1 of **Fig 1b**.

This capacitance C_1 is part of the antenna capacitance C and it's magnitude can be derived from the electric

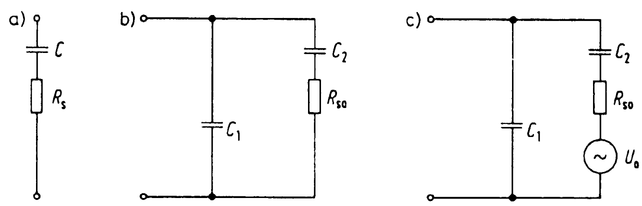

Fig 1. Equivalent circuits for the short radiator as a transmitting (a and b) and receiving (c) antenna.

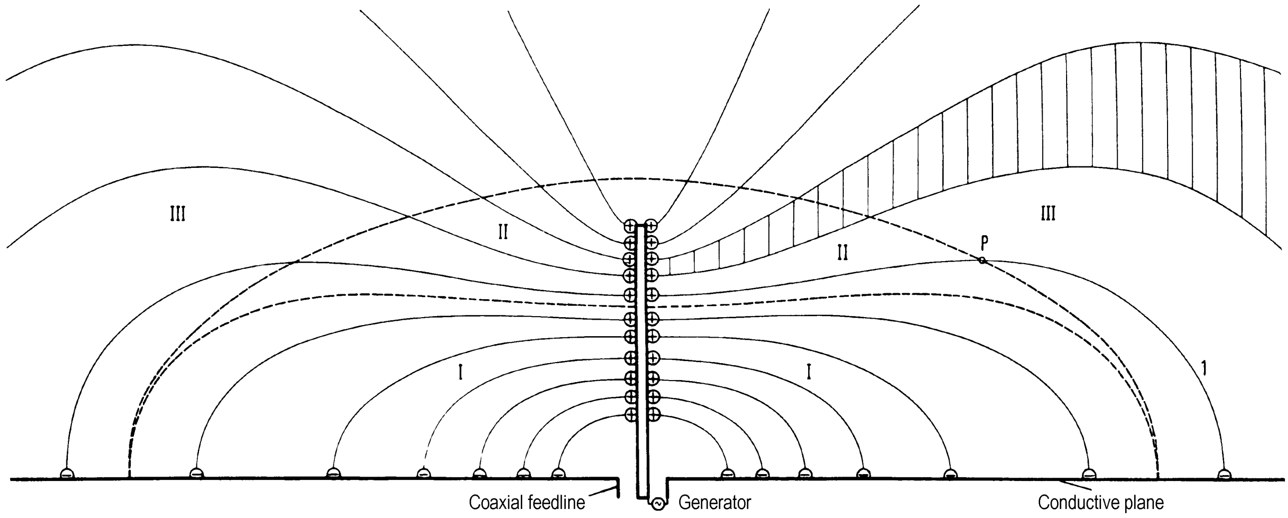

Fig 2. Electric field of a short rod antenna at the moment of maximum charge with 3 regions delimited by the broken lines; region I: dead capacitance C_1, region II: space capacitance C_2, region III: real resistance R_{S0}. A section between two field lines is shown hatched.

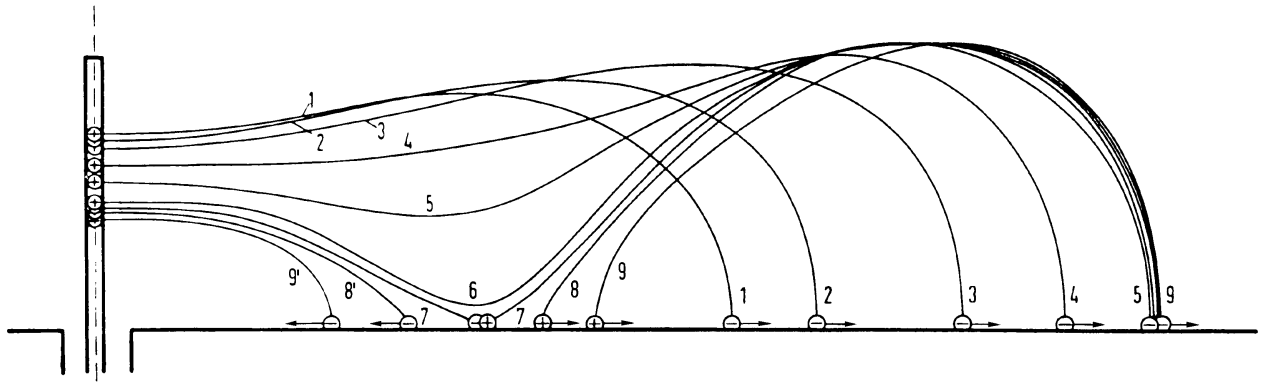

Fig 3. Temporal development and tearing up of a field line.

field of **Fig 2**. The field energy W_e contained in region I is

$$W_e = C_1 \, U_{max}^{\;2} / 2$$

where U_{max} is the voltage between the rod and the conductive plane at the moment of maximum voltage in **Fig 2** and which for short radiators is almost equal for all electric field lines. It follows that

$$C_1 = 2 \, W_e / U_{max}^{\;2} \tag{2}$$

and C_1 can be calculated from W_e and U_{max} as soon as the electric field of **Fig 2** is known [2].

The field lines outside of region I in **Fig 2** suffer a more complicated fate. During discharge, all the charges on the rod move down and back into the generator. The negative charges on the conductive plane are subject to two different influences: Firstly, the generator tries to pull them back to the feedpoint during discharge. Secondly, they are driven by their own inertia to continue their outward motion. For the charges on the plane which are outside of region I in **Fig 2** inertia predominates, because the pull of the generator decreases with increasing distance, so these charges are moving on outwards. Curve 1 in **Figs 2** and **3** shows the same momentary electric field line which is outside of region I. The other curves in **Fig 3** labeled with numbers in ascending order depict the temporal development of that field line during discharge. In the course of that and according to [2], the field line tears up by bending down at first (curves 4 to 6), then touching the plane (curve 7) and finally dividing into two parts (curves 8 and 8'). The part that sticks to the rod moves back to the generator (curve 9'), the other part travels out into space (curve 9). The arrows indicate the motional direction of the charges on the ground plane.

Fig 4 again shows curve 1 of **Figs 2** and **3**, a field line in the state of maximum antenna charge. Electric energy resides in the area of that field line. The curves of **Fig 4** emanating from that line show paths on which that energy subsequently flows. Parts of the energy flow back to the generator (curves *a*,*b*,*c*,*d*). They are delimited by the broken curve *e*, which emanates from a point P on field line 1. To the right of curve *e*, the energy paths lead into free space (curves *f*,*g*,*h*). All field lines which are outside of region I in **Fig 2** at the moment of maximum charge have such a characteristic point P. All these points together form the broken border line between regions II and III. That part of the energy of any field line that lies between it's point P and the rod in **Fig 2** flows back to the generator and represents reactive power which fills region II. That part that lies between point P and the ground plane flows out into space and fills region III. Looking at any sector between two neighbouring field lines in **Fig 2** (e.g. the hatched area), we can replace that part of the sector lying in region II by a storage capacitance and that part lying in region III by a real resistance. Then we get the equivalent circuit of **Fig 5a** for the impedance of the rod with parallel series combinations of R and C and with every series combination describing the impedance of one sector between two neighbouring field lines.

As long as the rod is short, the equivalent circuit of **Fig 5a** can be replaced by the simpler circuit of **Fig 5b**, which integrates the series capacitances of all sectors of region II into one capacitance C_2 and all real resistances into one single resistance R_{S0}.

The equivalent elements C_2 and R_{S0} can be derived according to the following procedure: **Fig 6** shows the broken border field line separating regions I and II. The voltage U between points A and B along that delimiting field line can be calculated if the field is known [2] by integrating the electric field strength along the field line. Rotation of that border line around the rod axis creates a surface separating regions I and II. At point A, where that surface meets the rod, the rod carries the current I_2. According to a generalized definition for impedances [6], $Z_2 = U / I_2$ is the impedance presented to that surface by the surrounding space. In **Fig 5a** this is the impedance of the parallel series combinations connected to points A and B and in **Fig 5b** this is the series combination of the equivalent capacitance C_2 and resistance R_{S0}. Because the current distribution on the rod [4] as well as the position of point A [2] are known, I_2 is also known and $Z_2 = U / I_2$ can be calculated.

Of particular interest is a more accurate knowledge of the real component R_{S0} which we calculate as follows: according to **Fig 5c**, R_S is the real component of the antenna impedance at the feedpoint of the radiator between terminals 1 and 2. P is the real power supplied by the generator and absorbed by the antenna, I_1 is the antenna current at the feedpoint. With a lossless dead capacitance C_1 the real power is

$$P = I_1^2 \, R_S / 2 = I_2^2 \, R_{S0} / 2 \qquad (3)$$

which is supplied to resistance R_S by current I_1 and to resistance R_{S0} by current I_2. It follows that like in [8]

$$R_{S0} = R_S \, (I_1 / I_2)^2. \qquad (4)$$

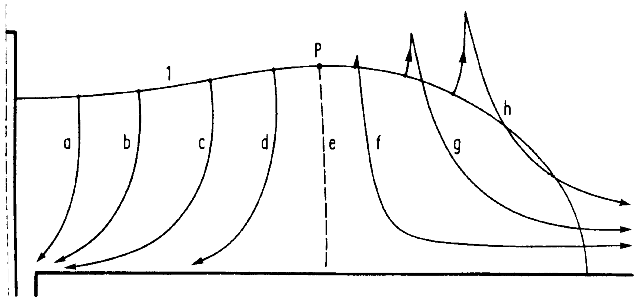

Fig 4. Paths of the energies which are located on field line 1 at the moment of maximum charge (Fig 2). Line *e* separates the energies which travel back to the generator during discharge from the energies which travel out into space.

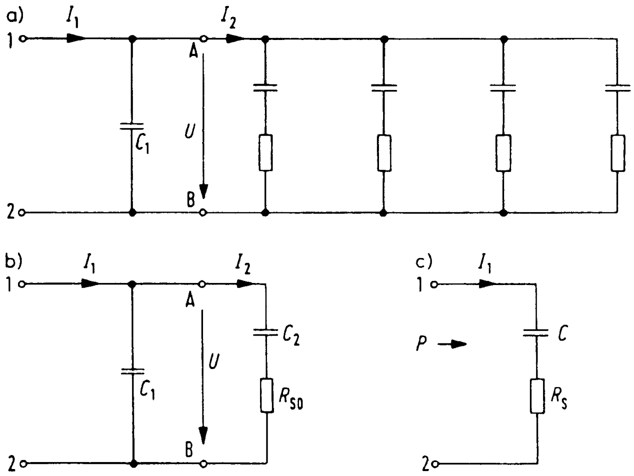

Fig 5. Equivalent circuits for the antenna impedance.

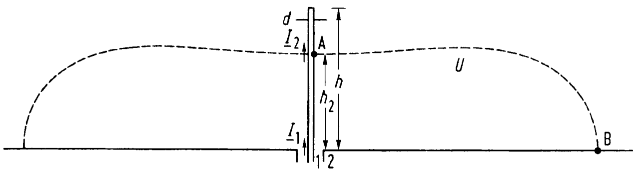

Fig 6. Border line between regions I and II.

R_S and the quotient I_1 / I_2 are known from [4] and then also R_{S0} from Eq (4). The interesting result of a large number of numerical evaluations is that within the limited computing accuracy

$$R_{S0} = 30 \pm 3 \ \Omega$$

which is almost independent of frequency and rod diameter. C_2 results with adequate accuracy assuming that in **Fig 5b** the reactance $1 / (\omega \, C_2)$ is considerably higher than R_{S0} and hence the current is approximately

$$I_2 = U \cdot \omega \, C_2 \qquad (5)$$

so that

$$C_2 = I_2 / (\omega \, U) \qquad (6)$$

is calculable from known quantities.

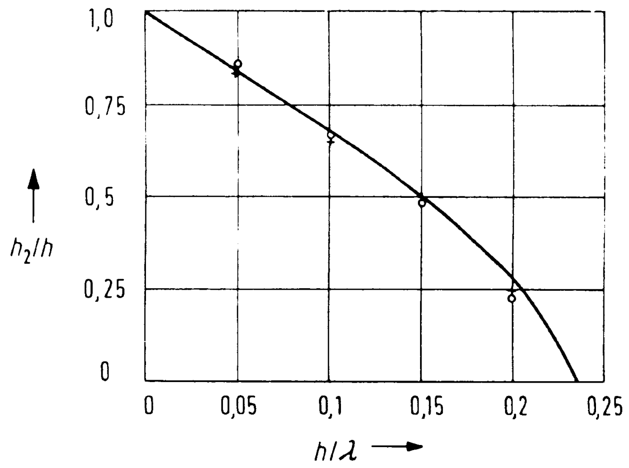

Fig 7. Height of the limiting point A on the rod; circles: h/d = 37, crosses : h/d = 130, solid line: theoretical graph assuming R_{S0}= 30 Ω.

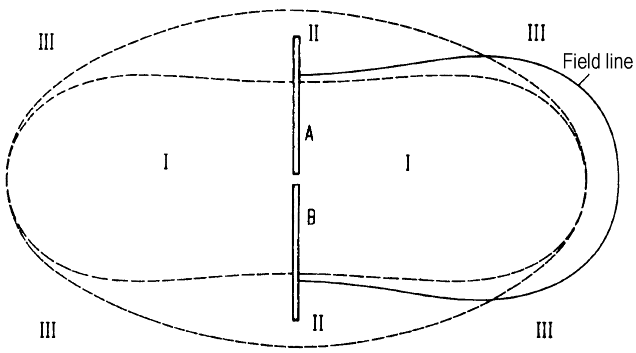

Fig 8. The three regions around a symmetrical dipole.

2. Frequency Dependence of the Equivalent Circuit

With h being the total antenna height and h_2 the height of point A on the delimiting field line in **Fig 6** our calculations showed that h_2 is almost independent of the rod diameter and decreasing with increasing frequency. **Fig 7** shows the values h_2 / h derived from the calculated field for h/d = 37 (circles) and h/d = 130 (crosses) with d being the rod diameter. For zero frequency ($h / \lambda_0 = 0$) $h_2 = h$, i.e. the whole rod represents dead capacitance, no wave detachment takes place and $C_1 = C$. The value h_2 / h decreases with increasing frequency so that for $h / \lambda_0 = 0.23$, i.e. at self-resonance of the rod (slightly below a quarterwave length of the rod), the whole rod radiates and $h_2 = 0$ [2]. The graph of **Fig 7** is based on the already mentioned value R_{S0} = 30 Ω and applies Eq (4) in the modified form

$$I_2 / I_1 = \sqrt{(R_S / R_{S0})}. \qquad (7)$$

R_S is known from [4] and with R_{S0} = 30 Ω we get the ratio I_2 / I_1 at point A. Because I_2 / I_1 can be obtained from [4] for any point on the rod, I_2 / I_1 calculated from Eq (7) yields the height h_2 of point A for R_{S0} = 30 Ω. The circles and crosses in **Fig 7**, which were calculated directly from the field (true h_2), are so very close to the solid line that the value R_{S0} = 30 Ω finds good confirmation.

Assuming static values for the antenna capacitance C of short rods as a first approximation yields in the case of small real components

$$C = C_1 + C_2 \qquad (8)$$

which is independent of frequency. The antenna capacitance C divides into a dead capacitance C_1 and a space capacitance C_2 and with increasing frequency C_1 is decreasing because of a decreasing h_2 according to **Fig 7** but C_2 is increasing. For the resonance frequency at $h / \lambda_0 = 0.23$, $h_2 = 0$ and hence $C_1 = 0$ and $C_2 = C$.

The delimiting field line between regions I and II beyond which wave detachment takes place is characterized by the feature that it's length is alwas slightly shorter than a quarter wavelength. As soon as the length of electric field lines in such a system becomes a quarter wavelength or more, they evidently are able to tear up and develop space waves. That kind of instability effect of longer field lines can also be noticed in many other electromagnetic wave processes and is possibly a basic law, which has not yet been formulated in general.

The rules found here for unsymmetrically fed rods can be applied also to symmetric dipoles by symmetric field supplementation. The equivalent circuits of **Fig 5** remain valid. **Fig 8** shows regions I to III for a dipole. Around the feedpoint there is region I which creates dead capacitance and the energy of which does not contribute to radiation. The length of the broken field line delimiting region I in **Fig 8** between A and B is slightly shorter than a half wavelength. So the tearing up of the symmetric field lines as they appear with dipoles obviously starts when they get longer than a half wavelength. **Fig 8** again shows the field line 1 from **Figs 2** and **3** at the moment of maximum charge of the dipole. That part of the field line which lies within region II becomes part of capacitance C_2, that part which lies within region III will tear up later and become part of the space wave and thus part of R_{S0}.

Though only a rod antenna has been examined so far, by means of the division of the field into regions I to III statements can be made also for other forms of antennas. Capacitance raising measures within region I, e.g. feedpoint insulators or metal parts in the proximity of the antenna, raise the dead capacitance C_1. Capacitance raising measures within region II, e.g. capacity hats, raise the space capacitance C_2.

Assuming that a radiation resistance R_S as high as possible is important for most applications of short antennas, the influence of dead capacitance C_1 on R_S can be calculated most simply by the approximate formula

$$R_S = R_{S0} \cdot (C_2 / C_1)^2. \qquad (9)$$

This equation is valid for the equivalent circuit of **Fig 5b** assuming that C_2 is considerably smaller than C_1 and R_{S0} is considerably smaller than the series connected $1 / (\omega C_2)$. Eq (9) shows that any increase of C_1 is detrimental and any increase of C_2 is favourable. These findings were confirmed by measurements carried out by the authors on short radiators embedded in a dielectric [7]. When the dielectric filled region I only, R_S became smaller than in air; when it filled regions I and II, R_S remained almost constant. When the dielectric like in **Fig 9** did not fill region I but only parts of regions II and III, R_S increased.

We are also often interested in the effective height h_{eff} of the antenna which can be defined by Eq (1), calculated from R_S and refered to $R_{S0} = 30\ \Omega$ by Eq (9). Then we get

$$h_{eff} / \lambda_0 = \sqrt{[R_S / (160\ \pi^2)]} = (C_2 / C_1) \sqrt{[R_{S0} / (160\ \pi^2)]} = 0.138\ (C_2 / C_1). \qquad (10)$$

This formula also shows that an increase of dead capacitance C_1, i.e. capacitance raising measures at the lower part of the rod, decreases the effective height, whereas an increase of space capacitance C_2, i.e. capacitance raising measures at the higher part of the rod, increases the effective height. Measures to increase C_2 were investigated in [5] and it has been shown that by these measures the frequency dependence of impedance decreases also for short antennas. Eq (10) yields

$$C_2 / C_1 = 7.26\ (h_{eff} / \lambda_0). \qquad (11)$$

Because it is known that for short rod antennas h_{eff} is almost independent of frequency while λ_0 is inversely proportional to frequency, it follows from Eq (11) that the quotient C_2 / C_1 is almost exactly proportional to frequency. With somewhat longer rods or higher frequencies the impedances are also influenced by the inductive effect of the magnetic field energy, which is always present in moving fields. This can be considered in first approximation by adding two inductances L_1 and L_2 to the equivalent circuit as shown in **Fig 10**. This equivalent circuit is suffice for all frequencies below self-resonance ($h / \lambda_0 = 0.23$). In the case of resonance, the capacitances of the antenna are compensated by the inductances L_1 and L_2 so that the impedance of the radiator becomes real. It is known that this

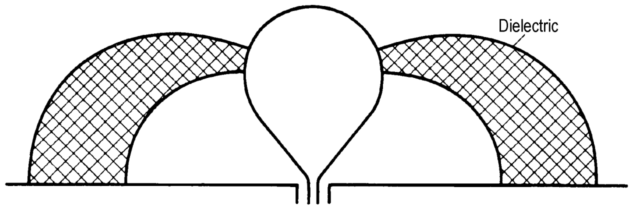

Fig 9. Enlargement of space capacitance by a dielectric body.

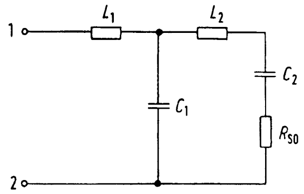

Fig 10. Equivalent circuit with inductances.

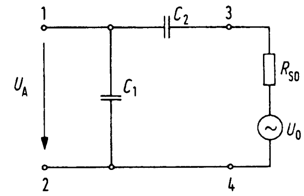

Fig 11. Equivalent circuit for the receiving antenna.

resistance has a value of approximately 35 Ω and is almost independent of the rod diameter. That strongly supports the finding that in the equivalent circuit of **Fig 5b** R_{S0} is approximately 30 Ω and almost independent of rod diameter and frequency. The fact that in case of resonance a real resistance of 35 Ω is met, slightly higher than $R_{S0} = 30\ \Omega$, can be explained by the resistance transformation of the equivalent circuit of **Fig 10** and additionally by the fact that the resonant rod antenna exhibits a more pronounced vertical directivity than the short rod. So the coupling to space and hence also R_{S0} for resonant rods is somewhat different than for short rods.

3. Equivalent Circuit for the Receiving Antenna

The effect of a wave received from the surrounding space can be modeled as a voltage source U_0 in series with R_{S0} according to **Fig 11**. If C_1 and C_2 are lossless, the maximum available power from that source is

$$P_{max} = U_0^{\,2} / (8\ R_{S0}) = U_A^{\,2} / (8\ R_S) \qquad (12)$$

with U_A being the unloaded voltage of the antenna across it's terminals 1 and 2. With the electric field strength at the antenna E and the antenna's effective height h_{eff} we get from known theories $U_A = E \cdot h_{eff}$. Then, assuming that $1 / (\omega C_2) >> R_{S0}$ and $C_2 << C_1$, according to Eqs (9) and (11) because of the capacitive voltage division of the source in **Fig 11** the unloaded voltage is approximately

$$U_0 = U_A\ (C_1 / C_2) = E\ h_{eff} \cdot (C_1 / C_2). \qquad (13)$$

If with short antennas C_1 / C_2 from Eq (11) is proportional to $1 / f$, the voltage U_0 caused by the wave action on the antenna is inversely proportional to the frequency f. Concerning frequency dependence, this corre-

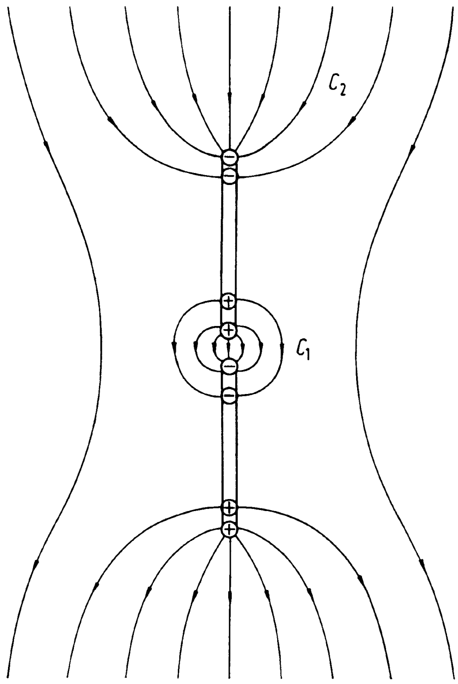

Fig 12. Electric field lines of a receiving dipole at the moment of maximum charge.

sponds to the kown formula for the effective area

$$A_{eff} = 3\ \lambda_0^{\,2} / (16\ \pi) \qquad (14)$$

of a short antenna which is inversely proportional to the squared frequency, since the power avalilable from the antenna is

$$P_{max} = U_0^{\,2} / (8\ R_{S0}) = S\ A_{eff} \qquad (15)$$

with

$$S = E\ H / 2 = E^{\,2} / (2\ Z_{F0}) \qquad (16)$$

being the radiation density of the received plane wave and Z_{F0} = 120 π Ω the characteristic impedance of space. Combining Eqs (12) to (16) with R_{S0} = 30 Ω we get the unloaded voltage of the equivalent voltage source of **Fig 11** as

$$U_0 = \sqrt{(S\ A_{eff} \cdot 8\ R_{S0})} = 0.195\ \lambda_0\ E. \qquad (17)$$

It follows that with R_{S0} being constant U_0 is independent of the length and diameter of the antenna rod. According to Eq (13), the unloaded voltage U_A of the short antenna across it's terminals 1 and 2 is

$$U_A = E \cdot h_{eff} = U_0\ C_2 / C_1 = E \cdot 0.138\ \lambda_0\ C_2 / C_1. \quad (18)$$

So it is possible to develop the notion that by the action of the receiving antenna on the arriving wave at first formally a voltage source, which is independent of the antenna dimensions, arises between terminals 3 and 4 in **Fig 11** with the unloaded voltage U_0 from Eq (17) and a source resistance of R_{S0} = 30 Ω. The antenna couples to that source by means of it's space capacitance C_2. The parallel dead capacitance C_1 then gives rise to the unloaded voltage U_A at the antenna feedpoint across terminals 1 and 2 according to Eq (18).

The equivalent circuit of **Fig 11** is confirmed by the antenna's field behaviour. Here the effect of the dead capacitance is best perceptible with the unloaded antenna, because then no load is present across terminals 1 and 2, i.e. parallel with dead capacitance C_1. The smaller the load impedance, the less effective is capacitance C_1. Just like with the transmitting antenna of **Fig 2**, the fields of the capacitances C_1 and C_2 are best perceptible in the state of maximum charge. **Fig 12** shows the momentary electric field of a dipole made up of two non-connected halfs in the field of a plane wave at the moment of maximum field strength. For short dipoles that field is practically identical to the electrostatic field which both halfs would create in a homogeneous field. The space capacitance can be recognized by the field lines extending from the dipole ends out into space and by which the dipole is coupled to the space wave. Also the dead capacitance can be recognized by the field lines between the dipole halfs.

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